

Consider a continuous-time signal that is a sum of two cosine signals observed for $-\infty < t < \infty$:

$$x(t) = \cos(2\pi f_0 t) + \cos(2\pi f_1 t)$$

where $f_1 = 10 f_0$. After sampling at a sampling rate $f_s > 2 f_0$, the discrete-time signal is

$$x[n] = \cos(\hat{\omega}_0 n) + 1$$

where

$$\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$$

and frequency f_0 does not alias; however, the frequency f_1 aliases.

What are all the possible values of f_s ?

Express your answers for f_s in terms of f_0 .

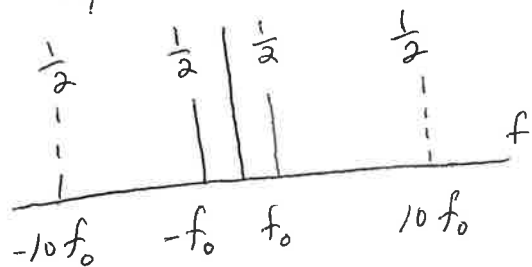
Answer:

$$x[n] = \cos(\hat{\omega}_0 n) + \cos(\hat{\omega}_1 n)$$

where $\hat{\omega}_1 = 2\pi \frac{f_1}{f_s}$. Since $\cos(\hat{\omega}_1 n) = 1$, $\hat{\omega}_1 \in \{0, 2\pi, 4\pi, \dots\}$.

So, frequency f_1 has, after sampling, aliased to 0 Hz. This means that $f_s \leq 2 f_1$, which is $f_s \leq 20 f_0$. Since frequency f_0 does not alias, $f_s > 2 f_0$. So, $2 f_0 < f_s \leq 20 f_0$.

Spectrum of $x(t)$



$$f_s \in \left\{ 10 f_0, \frac{10}{2} f_0, \frac{10}{3} f_0, \frac{10}{4} f_0 \right\}$$

Sampling has the effect on the frequency domain that each frequency component in $x(t)$ is present and also replicates at shifts equal to integer multiples of f_s . We find all four values of f_s that will create a replica of $f_1 = 10 f_0$ to land at the origin. Same for $-f_1$.